EFFECT OF INVENTORY TARGET AND REPLENISHMENT DELAY ON DYNAMIC EOQ

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Abstrak

Economic order quantity is a model used to calculate the optimal order for an item based on the cost of ordering, buying, storing and shortage. Therefore, we propose dynamic EOQ consisting of the variables, which have interrelation, delay factor, feedback loop, and as a closed system in the system dynamics model. Through this model, we can do various scenario to get the minimum total cost associated with policies that can be done on variables controlled (exogenous variable of a dynamic system). However, we only limit the two scenarios to get the minimum total cost, which is the inventory target and delay of replenishment. Based on two scenarios, this article can show the simulation output as the graph of variables (the dynamic behavior) and clarified by statistical data (maximal value, minimal value, total, deviation standard, and average) that can be used to choose the policy to achieve the minimum total cost of dynamic EOQ. In this article, scenario increasing inventory target results in the decrease of total cost less than the decreasing delay of replenishment.

Kata kunci: *Economic order quantity, total cost, dynamic, increasing inventory target, delay of replenishment.*

1. PRELIMINARY

Economic order quantity (EOQ) was first developed by F. W. Harris in 1915 [1] and published in The Magazine of Management [2]. This model is used to determine the minimum cost of order amount, inventory, shortage, and purchase (replenishment). Initially, this EOQ is applied to a constant demand for all time, and each order is sent full when inventory reaches zero.

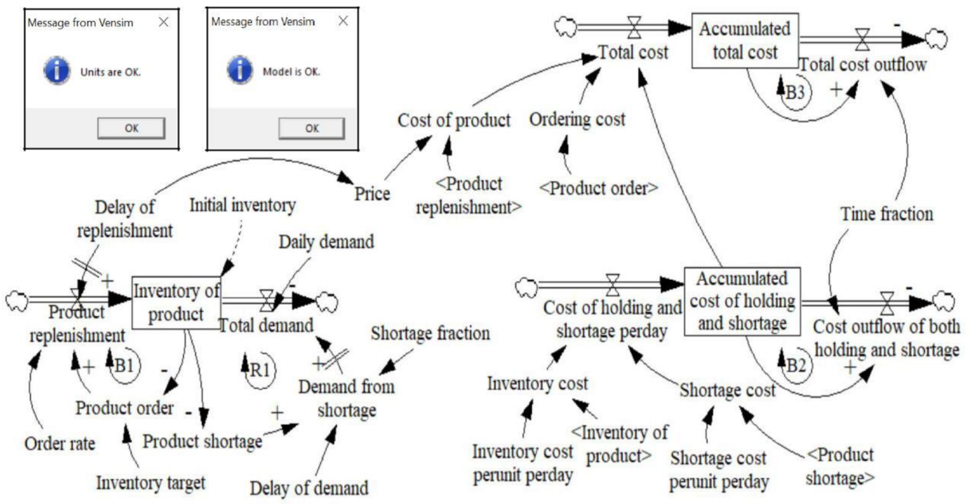
Until recently, EOQ has been widely studied and developed by many researchers. Therefore, we have reviewed the developed EOQ as follows. Initially, EOQ has extended to the case of variable setup numbers n and m for production and repair within some collection time interval by Richter in 1996 [3]. In the next development, EOQ has been purposed by to model the retailer's profit-maximizing strategy when confronted with supplier's trade offer of credit and price-discount on the purchase of merchandise by Sana and Chaudhuri in 2008 [4]. EOQ has used to determine the retailer's optimal order quantity for similar products. It is assumed that the amount of display space is limited and the demand of the products depends on the display stock level and the initiatives of sales staff where more stock of one product makes a negative impression of another product by Sana in 2012 [5]. The last few years, EOQ has introduced and examined as a generalized formula based on the model with linear and fixed back ordering costs by Sphicas in 2014 [6]. EOQ has developed to fuzzy inventory model with backorders that consider human learning over the planning horizon by Kazemi et al in 2015 [7]. EOQ has contributed by formulating the inventory problem for a wholesaler facing batch demands and using backorder to control the inventory by Huang and Wu in 2016 [8]. EOQ has extended to permits a minimal set of assumptions on the ordering/procurement and holding/backorder costs and establish necessary and sufficient conditions for the existence of an optimal policy of the (s, S) type by Chuang and Chiang in 2016 [9]. In the dynamical system, the study deals with an inventory model to determine the retailer's optimal order quantity for similar products that is done by Sana in 2012 [5].

Based on the articles we reviewed, there has been no researcher studying the dynamic EOQ. Where in EOQ dynamically, demand occurs continuously at a specific duration time that must be meet with the cost of ordering cost, shortage cost, cost of a product, and shortage cost. Therefore, we propose the development of EOQ dynamically by simulating two scenarios to that the aim is the achieving minimum total cost consisting of ordering cost, shortage cost, cost of a product, and shortage cost. The scenarios of dynamic EOQ are the increasing inventory target and the decreasing delay of replenishment. In this article, dynamic EOQ consists of interrelated variables as a component of a closed system and contain time delay and feedback loop. Based on the characteristic described above, the method that using to achieve minimum total cost on dynamic EOQ is system dynamics which is represented by the stock and flow diagram (SFD). Finally, this article shows the simulation output as the graph of variables and statistical data that can be used to choose the policy to achieve the minimum total cost of dynamic EOQ.

2. PROPOSED MODEL

Dynamic EOQ proposed in this article is a system dynamics model that has two demands, that is the daily demand generated from the uniform random number generator and the demand to serve back order that derives from the shortage of inventory. Where both demands are as the outflow variable that reduces the inventory of the product (as the stock variable). In this dynamic model, the increasing of product inventory (as the inflow variable) is caused by product order that has the delay. Product order occurs when the product inventory is less than or equal to zero. Furthermore, if there is an event of product shortage, then this shortage will be fulfilled by demand from delayed shortages.

To calculate the total cost, dynamic EOQ adds each ordering cost, cost of a product, inventory cost, and shortage cost during simulation time so that the value of total cost has a dynamic behavior during simulation. The cost of a product is calculated based on the quantity of product (product replenishment) multiplied by the price of a product while the holding cost and shortage costs are calculated based on the inventory of product multiplied by inventory cost per unit-day and product shortage multiplied by shortage cost per unit-day. In this dynamic EOQ, the total cost is an auxiliary variable that used as an inflow variable into the accumulated total cost (the stock variable). Next, the stock variable is added the total cost outflow so that this model forms a closed system [10][11]. Therefore, the system dynamics model of dynamic EOQ is represented by SFD as shown in Fig. 1. The SFD consists of variables, which are stock, auxiliary, inflow, outflow, and constant (exogenous variable) as described in Table 1. The definition of each variable in this table shows the algorithms of the system dynamics model. In this Table, there are additional variables (order rate, shortage fraction, and time fraction) which are used to form an equation with the right unit on the stock variables [12][13][14].



Gambar 1. Model of EOQ dynamically

Table 1. Definition of variables

Variable name	Definition	Type	Units
Accumulated total cost	INTEG (Total cost-Total cost outflow,0)	Stock	dollar*day
Cost of holding and shortage	INTEG (Cost of holding and shortage perday,0)	Stock	Dollar
Cost of holding and shortage perday	Inventory cost+Shortage cost	Auxiliary (inflow)	Dollar/day
Cost of product	Price*Product replenishment	Auxiliary	Dollar
Cost outflow of both holding and shortage	Accumulated cost of holding and shortage *Time fraction	Auxiliary (outflow)	dollar/Day
Daily demand	INTEGER(RANDOM UNIFORM (0,10,1))	Constant	unit/day
Delay of demand	2	Constant	day
Delay of replenishment	3	Constant	day
Demand from shortage	DELAY FIXED(Product shortage*Shortage fraction, Delay of demand,0)	Auxiliary	unit/day
Initial inventory	50	Constant	unit
Inventory cost	IF THEN ELSE(Inventory of product>0,Inventory of product,0)*Inventory cost per unit per day	Auxiliary	dollar/Day
Inventory cost per unit per day	2	Constant	dollar/(unit*Day)
Inventory of product	INTEG (Product replenishment-Total demand, Initial inventory)	Stock	unit
Inventory target	25	Constant	unit
Order rate	1	Constant	1/day
Ordering cost	IF THEN ELSE(Product order>1,0.25,0)	Auxiliary	dollar
Price	IF THEN ELSE(Delay of replenishment <=1, 30, IF THEN ELSE(Delay of replenishment<=2,20,40))	Constant	dollar*(day/unit)
Product order	IF THEN ELSE(Inventory of product < Inventory target, Inventory target-Inventory of product,0)	Auxiliary	unit
Product replenishment	DELAY FIXED(Product order*Order rate, Delay of replenishment,0)	Auxiliary (Inflow)	unit/Day
Product shortage	ABS(IF THEN ELSE(Inventory of product<=0,Inventory of product,0))	Auxiliary	unit
Shortage cost	Product shortage*Shortage cost per unit per day	Auxiliary	dollar/day
Shortage cost per unit per day	4	Constant	dollar/(unit*Day)
Shortage fraction	1	Constant	1/Day
Time fraction	1	Constant	1/day
Total cost	Accumulated cost of holding and shortage + Cost of product + Ordering cost	Auxiliary (Inflow)	dollar
Total cost outflow	Accumulated total cost*Time fraction	Auxiliary (Outflow)	dollar

2.1 *Testing Model*

This dynamic EOQ is not tested by the behavioral testing as explained by Barlas [15] because this model is not developed using reference data for one of the key variables in the model. However, this dynamic EOQ is conducted structural testing that consists of unit testing to ensure the accuracy of the unit of all variables and model testing to ensure the model has formed a closed system [16, 17]. Therefore, the SFD diagram involves the information dialogue ("Units are OK and Model is OK") that shows the dynamic EOQ model has been tested by the structural testing. In this SFD diagram, there are no terminate variable so the dynamic EOQ model forms a closed system and contains four feedback loops; three negative loops (B1, B2, and B3) and one positive loop (R1).

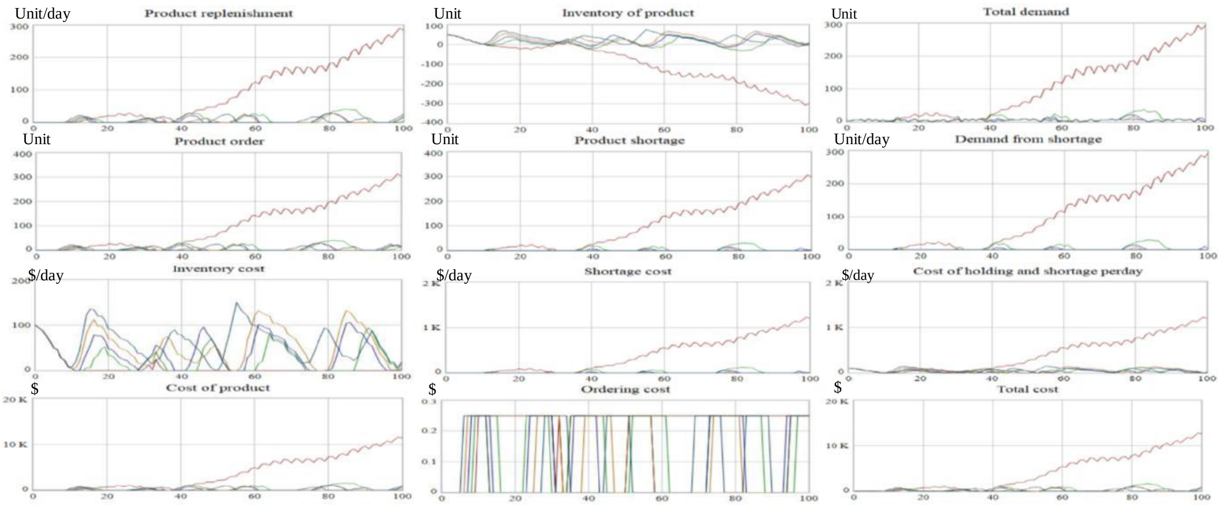
- Inventory of product $\xrightarrow{(-)}$ Product order $\xrightarrow{(+)}$ Product replenishment $\xrightarrow{(+)}$ Inventory of product: B1.
- Inventory of product $\xrightarrow{(-)}$ Product shortage $\xrightarrow{(+)}$ Demand from shortage $\xrightarrow{(+)}$ Total demand $\xrightarrow{(-)}$ Inventory of product: R1.
- Accumulated cost of holding and shortage $\xrightarrow{(+)}$ Cost outflow of both holding and shortage $\xrightarrow{(-)}$ Accumulated cost of holding and shortage: B2.
- Accumulated total cost $\xrightarrow{(+)}$ Total cost outflow $\xrightarrow{(-)}$ Accumulated total cost: B3.

2.2 Scenario and Simulation

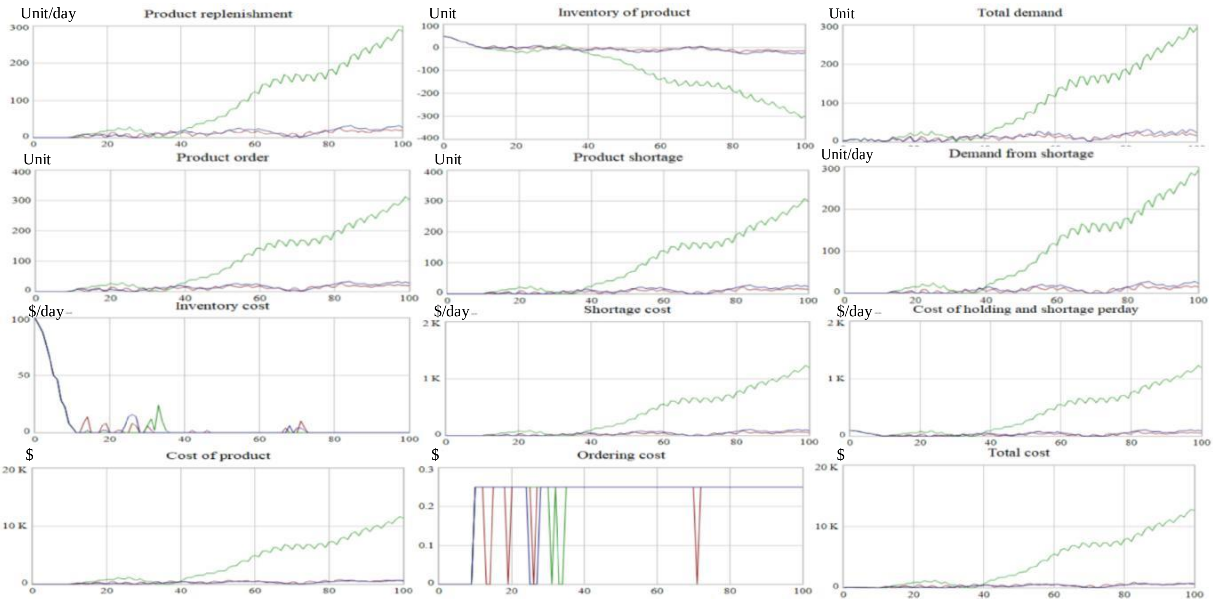
To achieve the minimum total cost, dynamic EOQ model is simulated with two scenarios. The first scenario is the increase of inventory target ( ) and the fixed delay ( ) of replenishment (= 5, 10, 15, 20 , and 25 units, = 3 days). Next, the second scenario is the decrease of the delay in replenishment and the fixed inventory target (= 2 and 1 days for = 5 unit).

3. RESULT AND DISCUSSION

The outputs of the first scenario simulations are given in Fig. 2 (for the scenario of increasing) and Fig.3 (for the scenario of decreasing). Next, the outputs are summarized to the maximum value ( ), the minimum value ( ), total value during simulation time ( ), standard deviation ( ), and average ( ) as shown in Table 2.



Gambar 2. Simulation output that is the change in dynamic behavior due to the scenario of increasing inventory targets



Gambar 3. Simulation output that is the change in dynamic behavior due to the scenario of decreasing replenishment delay

The output of the first scenario simulation shows the graphs forming an oscillation except ordering cost. It is due to the negative feedback (B1, B2, and B3) and the delay [10] in the model. Especially in the inventory of products, there are two feedback loops, which are opposite, B1 and R1. In this scenario, if the higher the then the total cost will decrease. The minimum total cost in = 25 with = 3 (*TTotal cost* reaches 24332.5 dollars). While changed of the dynamic behavior of other variables can be seen from the statistical data of two outputs of scenario simulation, that are *Vmax*, *Vmin*, *T*, *DS*, and *A* (Table 2). In general, all variables decreased except the inventory cost and cost of holding and shortage per day [5]. Next, the output of the second scenario simulation also shows the graphs that form an oscillation except ordering cost. In this scenario, the smallest total cost is not occurred at = 1 but at = 2, which is 30606 dollars. In both scenario simulations, the scenario of decreased inventory target is still better than the increased delay of replenishment.

Table 2. Statistical data of two outputs of scenario simulation

Variable	Statistical Data	Scenario 1					Scenario 2	
		IT = 5 *DR* = 3	*IT* = 10 *DR* = 3	*IT* = 15 *DR* = 3	*IT* = 20 *DR* = 3	*IT* = 25 *DR* = 3	*DR* = 2 *IT* = 5	*DR* = 1 *IT* = 5
Product replenishment	V_{Min} (Unit/day)	0	0	0	0	0	0	0
	V_{Max} (Unit/day)	291	40	28	28	24	33	27
	T (Unit/day)	9468	874	572	480	458	1312	1096
	DS	91.51	11.77	8.35	7.31	6.94	9.68	7.07
	A (Unit/day^2)	93.74	8.65	5.66	4.75	4.53	12.99	10.85
Inventory of product	V_{Min} (Unit)	-308	-30	-13	-8	1	-28	-22
	V_{Max} (Unit)	50	50	53	66	75	50	50
	T (Unit)	-9630	825	1866	2670	3010	-620	-369
	DS	99.72	19.59	18.39	19.81	17.77	15.37	13.29
	A (Unit/day)	-95.35	8.17	18.48	26.44	29.80	-6.14	-3.65
Total demand	V_{Min} (Unit)	1	0	0	0	0	0	0
	V_{Max} (Unit)	294	37	21	16	9	33	25
	T (Unit)	9825	925	600	505	484	1381	1156
	DS	92.56	8.54	4.29	3.21	2.84	8.99	6.44
	A (Unit/day)	97.28	9.16	5.94	5.00	4.79	13.67	11.45
Product order	V_{Min} (Unit)	0	0	0	0	0	0	0
	V_{Max} (Unit)	313	40	28	28	24	33	27
	T (Unit)	10382	901	622	534	512	1371	1115
	DS	96.76	11.69	8.51	7.63	7.27	9.77	7.03
	A (Unit/day)	102.79	8.92	6.16	5.29	5.07	13.57	11.04
Product shortage	V_{Min} (Unit)	0	0	0	0	0	0	0
	V_{Max} (Unit)	308	30	13	8	0	28	22
	T (Unit)	9948	445	119	23	0	946	701
	DS	96.02	8.11	2.91	1.10	0	8.81	5.81
	A (Unit/day)	98.50	4.41	1.18	0.23	0	9.37	6.94
Demand from shortage	V_{Min} (Unit/day)	0	0	0	0	0	0	0
	V_{Max} (Unit/day)	292	30	13	8	0	28	22
	T (Unit/day)	9341	441	116	21	0	897	672
	DS	92.40	8.13	2.90	1.09	0	8.63	5.79
	A (Unit/day^2)	92.49	4.37	1.15	0.21	0	8.88	6.65
Inventory cost	V_{Min} ($/day)	0	0	0	0	2	0	0
	V_{Max} ($/day)	100	100	106	132	150	100	100
	T ($/day)	636	2540	3970	5386	6020	652	664
	DS	19.82	28.71	33.65	38.92	35.53	19.74	19.67
	A ($/day^2)	6.30	25.15	39.31	53.33	59.60	6.46	6.57
Shortage cost	V_{Min} ($/day)	0	0	0	0	0	0	0
	V_{Max} ($/day)	1232	120	52	32	0	112	88
	T ($/day)	39792	1780	476	92	0	3784	2804
	DS	384.09	32.46	11.63	4.41	0	35.24	23.23
	A ($/day^2)	393.98	17.62	4.71	0.91	0	37.47	27.76
Cost of holding and shortage per day	V_{Min} ($/day)	2	0	0	0	2	0	0
	V_{Max} ($/day)	1232	120	106	132	150	112	100
	T ($/day)	40428	4320	4446	5478	6020	4436	3468
	DS	378.03	31.34	29.89	37.90	35.53	33.80	23.62
	A ($/day^2)	400.28	42.77	44.02	54.24	59.60	43.92	34.34
Cost of product	V_{Min} ($)	0	0	0	0	0	0	0
	V_{Max} ($)	11640	1600	1120	1120	960	660	810
	T ($)	378720	34960	22880	19200	18320	26240	32880
	DS	3660.27	470.66	334.06	292.38	277.51	193.57	211.98
	A ($/day)	3749.70	346.14	226.53	190.10	181.39	259.80	325.54
Ordering cost	V_{Min} ($)	0	0	0	0	0	0	0
	V_{Max} ($)	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	T ($)	22	13.25	11	10.5	10.5	22	21.5
	DS	0.08	0.13	0.12	0.12	0.12	0.08	0.09
	A ($)	0.22	0.13	0.11	0.10	0.10	0.22	0.21
Total cost	V_{Min} ($)	0	0	0	0	0	0	0
	V_{Max} ($)	12808.3	1712.25	1160.25	1122.25	1010	752.25	898.25
	T ($)	417974.6	39281.25	27333	24674.5	24332.5	30606	36313.5
	DS	4028.93	481.99	322.62	275.24	261.11	219.50	225.69
	A ($/day)	4138.36	388.92	270.62	244.30	240.92	303.03	359.54

4. CONCLUSION AND FUTURE WORK

This article has proposed dynamic EOQ model that can be used to simulate two scenarios to achieve minimum total cost with system dynamics. The scenarios in this model are the increasing inventory target and the decreasing delay of replenishment. In this dynamic EOQ model, there are no terminate variable in the model so that it forms a closed system, which has four feedback loops. The output of both scenario simulations show graphs that form an oscillation, which due to the negative feedback and the delay.

Based on the result of two scenarios, the increasing inventory target results in the decrease of total cost less than the decreasing delay of replenishment. Furthermore, the increasing inventory target scenario causes the decrease in the most variables on dynamic EOQ model as shown in Table 2. While the scenario of decreasing delay of replenishment causes the decrease and increase in the variables of dynamic EOQ model. Even, in this scenario, the decrease in the delay of replenishment actually causes the increase in total cost.

In the future work, we will change another variable in this dynamic EOQ so that we know the effect of these decreasing variables to total cost. This dynamic EOQ also can be developed in the way add new variables for enlarging a problem of a system that related inventory system.

5. REFERENCES

[1] Erlenkotter, D., 1989. Note—An Early Classic Misplaced: Ford W. Harris's Economic Order Quantity Model of 1915. *Management Science,* 35 (7) pp.771-902.

[2] Harris, P. W., 1913. How many parts to make at once. *The Magazine of Management,* 10 (2), pp.135-126.

[3] Richter, K., 1996. The EOQ repair and waste disposal model with variable setup numbers. *European Journal of Operational Research,* 95 (2), pp.313-324.

[4] Sana, S. S. and Chaudhuri, K. S., 2008. A deterministic EOQ model with delays in payments and price-discount offers. *European Journal of Operational Research,* 184 (2), pp.509-533.

[5] Sana, S. S., 2012. The EOQ model – A dynamical system. *Applied Mathematics and Computation,* 218 (17) pp.8736-8749.

[6] Sphicas, G. P., 2014. Generalized EOQ formula using a new parameter: Coefficient of backorder attractiveness. *International Journal of Production Economics,* 155 pp.143-147.

[7] Kazemi, N., Shekarian, E., Cárdenas-Barrón, L. E. and Olugu, E. U., 2015. Incorporating human learning into a fuzzy EOQ inventory model with backorders. *Computers & Industrial Engineering,* 87 pp. 540-542.

[8] Huang, B. and Wu, A., 2016. EOQ model with batch demand and planned backorders," *Applied Mathematical Modelling,* 40 (9-100), pp. 5482-5496.

[9] Chuang, C. -H. and Chiang, C. -Y., 2016. Dynamic and stochastic behavior of coefficient of demand uncertainty incorporated with EOQ variables: An application in finished-goods inventory from General Motors′ dealerships. *International Journal of Production Economics,* vol. 172, pp. 95-109.

[10] Sterman, J. D. , Business Dynamics: Systems Thinking and Modeling for a Complex World., Boston: McGraw-Hill Higher Education., 2000.

[11] Elsawah, S., Pierce, S. A., Hamilton, S. H. and Delden, H. v., 2017. An overview of the system dynamics process for integrated modelling of socio-ecological systems: Lessons on good modelling practice from five case studies. *Environmental Modelling & Software,* 93, pp. 127-145.

[12] Lee, S., Han, W. and Park, Y., 2015. Measuring the functional dynamics of product-service system: A system dynamics approach. *Computers & Industrial Engineering,* 80, pp. 159–170.

[13] Thompson, J. P., Howick, S and Belton, V., 2016. Critical Learning Incidents in system dynamics modellingen gagements," *European Journal of Operational Research,* 249, pp. 945–958.

[14] Langroodi, R. R. P. and Amiri, M., 2016. Asystem dynamics modeling approach for a multi-level, multi-product, multi-region supply chain under demand uncertainty. *Expert Systems With Applications,* 51, pp. 231–244.

[15] Barlas, Y., 1989. Multiple tests for validation of system dynamics," *European Journal of Operational Research,* 42, (1), pp. 59-87.

[16] Barlas, Y., 1996. Formal aspects of model validity and validation in system dynamics, *System Dynamic Review,* vol. 12, (3), pp. 183-210.

[17] Qudrat-Ullah, H. and Seong, B. S., 2010. How to do structural validity of a system dynamics type simulation model: The case of an energy policy model. *Energy Policy,* 38, pp.2216–2224.

[18] Sana, S. S. , 2012. The EOQ model – A dynamical system. *Applied Mathematics and Computation,* 218, pp.8736–8749.